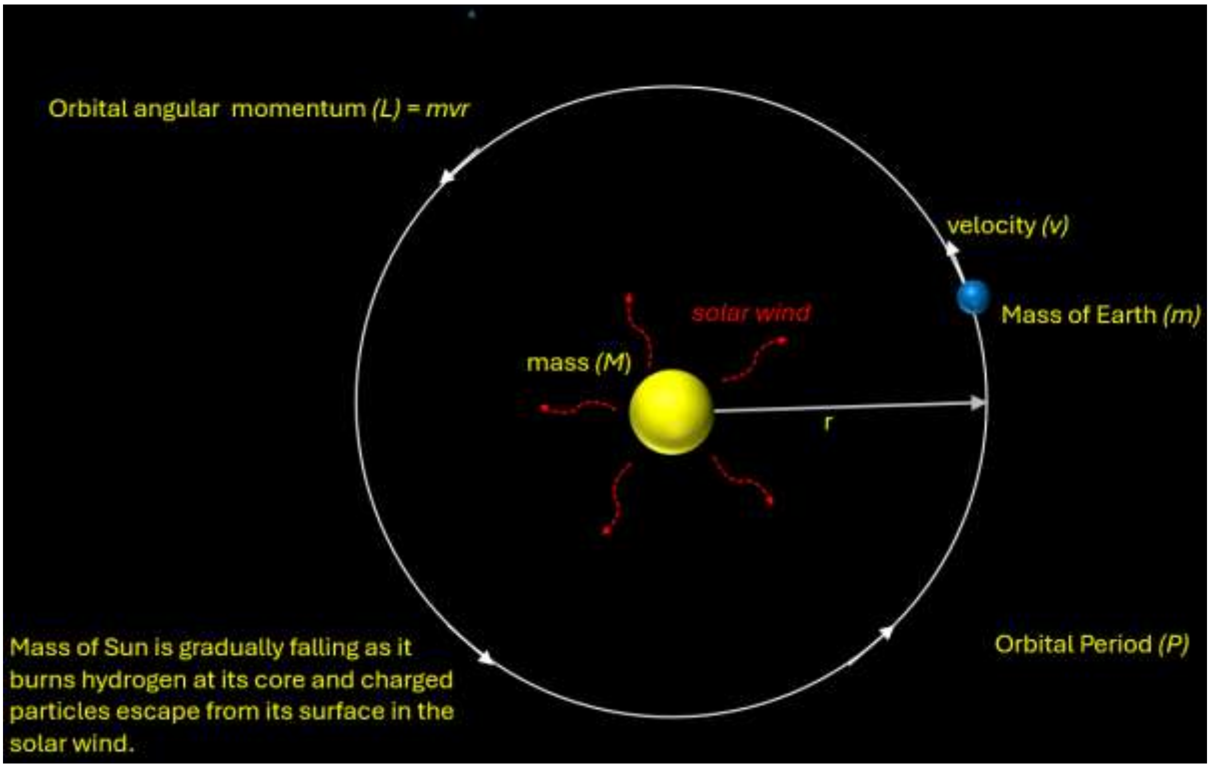
Changes in Earths Orbital Parameters as result of Solar Mass Loss

This paper gives a simple derivation of the formulae to determine

- The rate of change ($\frac{dr}{dt}$) of the radius of the Earth's orbit around the Sun *(r)*.
- The rate of change ($\frac{dv}{dt}$) of the velocity of the Earth's in its orbit *(v)*.
- The rate of change ($\frac{dP}{dt}$) of the period of the Earth's orbit *(P)*.

due to the Sun's mass loss caused by solar wind from its surface and fusion at its core.

For simplicity it is assumed that the Earth's orbit around the Sun is circular, in which case the rules of orbital mechanics means it must move with a constant velocity.



1 Fundamental Principal Conservation of Angular Momentum

As the Sun loses mass its gravitational pull on the Earth weakens, as a result the Earth spirals out to a higher orbit, its orbital speed slows and its orbital period lengthens. To calculate the magnitude of these changes we use the fact that the changes to the Earth's orbit obey the *law of conservation of angular momentum*.

This states:

"*The total angular momentum of a system (e.g. the Earth orbiting the Sun) remains constant if no external torque (i.e. a twisting force) acts upon it*"

2 Derivation of the Formula for the Rate of change of Orbital Radius

The Earth's orbital angular momentum *L* is given by:

$$L = mv(t)r(t) \qquad (1a)$$

In addition, for a body (the Earth) in circular orbit around a much more massive body (the Sun) the orbital velocity is given by:

$$v(t)^2 = \frac{GM(t)}{r(t)} \qquad (1b)$$

Therefore $$L = m\sqrt{GM(t)r(t)} \qquad (1c)$$

Where:

- *m* is the mass of the Earth
- *G* is Newton's gravitational constant
- *M(t)* is the mass of the Sun which gradually changes with time
- *r(t)* is the Earth's orbital radius which gradually changes with time
- *v(t)* is the Earth's orbital velocity which gradually changes with time

Since *L, m* and *G* are constants $\sqrt{M(t)r(t)}$ must also be a constant. So, equation (1c) can be written:

$\sqrt{M(t)r(t)} = C_1$ (2)

Squaring the above equation gives:

$M(t)r(t) = C_1^2$ (3)

Differentiating (3) with respect to time*(t),* using the product rule gives:

$$\frac{dM}{dt}r(t) + \frac{dr}{dt}M(t) = 0 \qquad (4)$$

Rearranging to make $(\frac{dr}{dt})$ the subject and, for conciseness, using a dot to denote the derivative with respect to time *(t)* and writing *r* for *r(t)*, *M* for *M(t)* gives:

$$\dot{r} = -r\,(\frac{\dot{M}}{M}) \qquad (5)$$

3 Derivation of the Formula for the Rate of Change of Orbital Velocity

If we take equation (1a), and substitute for *r(t)* using $r(t) = GM/v(t)^2$ from equation (1b), then we get:

$$L = \frac{mGM(t)}{v(t)} \qquad (6)$$

Since *L, m* and *G* are constants $M(t)/v(t)$ must also be a constant. So, equation (6) can be written:

$$\frac{M(t)}{v(t)} = C_2 \qquad (7)$$

Differentiating (7) with respect to time using the quotient rule and, for conciseness, using a dot to denote the time derivative *and* writing *v* for*(t)*, *M* for *M(t)* gives:

$$\frac{1}{M^2}(\dot{M}v - M\dot{v}) = 0^{\cdot} \qquad (8)$$

Rearranging (8) gives:

$$\dot{v} = v\frac{\dot{M}}{M} \qquad (9)$$

4 Derivation of the Formula for the Rate of Change of Orbital Period

For a circular orbit the orbital period is given by:

$$P^2 = \frac{4\pi^2 r(t)^3}{GM} \qquad (10)$$

Rearranging equation (3) which we derived previously we get $r(t) = \frac{C_1^2}{M(t)}$ where C_1 is a constant; so (10) becomes:

$$P(t)^2 = \frac{4\pi^2 C_1^2}{GM(t)^4} \qquad (11)$$

Taking the square root of the above equation gives us:

$$P(t) = \frac{C_3}{M(t)^2} \qquad (12)$$

where C_3 is a constant.

Differentiating (12) with respect to time, using the chain rule, and for conciseness, using a dot to denote the time derivative and writing *P* for *P(t)*, *M* for *M(t)* gives:

$$\dot{P} = \frac{-2C_3\,\dot{M}}{M^3} \qquad (13)$$

Combining equations (12) and (13) gives:

$$\dot{P} = -2P\frac{\dot{M}}{M} \qquad (14)$$

5 Calculation of the Rates of Change

To calculate the actual rate of change we will assume the following.

- The rate of mass change $\dot{M}$ of the Sun is -5.6 x 10^9 kg s^{-1} which is -1.77 x 10^{17} kg $year^{-1}$. *The negative sign indicates that mass is being lost*.
- The total mass *(M)* of the Sun is 1.99 x 10^{30} kg.

  ➔the fractional solar mass change ($\frac{\dot{M}}{M}$) is -8.89 x 10^{-14} $year^{-1}$

- The radius of the Earth's orbit is 1.50 x 10^{11} m.
- The orbital velocity is 29800 m s^{-1}.

- The orbital period is 365.256393 days (= 3.16×10^7 s).

Using the previous formulae, we obtain:

Change in orbital radius ($\dot{r}$) - from equation (5)

$\dot{r} = 1.50 \times 10^{11}$ m X –(-8.89 x 10^{-14} year^{-1}) = 0.0133 m year^{-1}

The Earth's orbital radius is increasing at a rate of approximately 1.33 cm per year.

If we look over the next 10 million years, $\dot{M}$ will not change significantly, neither will ($\frac{\dot{M}}{M}$) since the value of $\dot{M}$ is extremely small in relation to the value of M. So, the orbital radius will increase by approximately 133 km.

Change in orbital velocity ($\dot{v}$) - from equation (9)

$\dot{v} = 29800$ m s^{-1} X (-8.89 x 10^{-14} year^{-1}) = - 2.65 x 10^{-9} (m s^{-1}) year^{-1}

The Earth's orbital velocity is slowing down at a rate of approximately 2.65 nanometres per second per year. If we look over the next 10 million years the orbital velocity will decrease by approximately 2.65 cm per second (0.0936 km/h).

Change in orbital Period ($\dot{P}$) - from equation (14)

$\dot{r} = 3.16\text{x}10^7$ s X –2(-8.89 x10^{-14} year^{-1}) = 5.61 x 10^{-6} s year^{-1}

The Earth's orbital velocity is period is increasing at a rate of approximately 5.61 microseconds per year. If we look over the next 10 million years the orbital period will increase by approximately 56.1 seconds (just under a minute).